Pergamon

Int. J. Solids Structures Vol. 34, No. 12, pp. 1505–1516, 1997

0020–7683/97 $17.00 + .00

PII: S0020–7683(96)00091–1

FRICTIONAL HEATING IN SLIDING CONTACT OF TWO THERMOELASTIC BODIES

V. J. PAUK and A. A. YEVTUSHENKO
Department of Mechanics and Mathematics, Lviv University, 1 Universitetska St., 290602 Lviv, Ukraine

(*Received* 21 *March* 1995; *in revised form* 9 *May* 1996)

Abstract—The plane contact problem of an elastic thermally conducting cylinder sliding over the surface of a thermoelastic half-space is considered. The problem is analysed with the convective cooling of the cylinder surface in separation region and non-ideal thermal contact between bodies. The problem is formulated in terms of three governing integral equations with unknown pressure and contact temperatures. The equations are solved numerically, using the Gauss–Jacobi and trapezoidal-rule quadratures. The effect of Peclet's number, Biot's number and the thermal conductivities of the temperature fields and the load required to maintain the contact path at a fixed value are investigated. © 1997 Elsevier Science Ltd. All rights reserved.

INTRODUCTION

Plane contact problems involving frictional heating for a cylinder sliding over the surface of the half-space have been considered by many authors (see e.g. Lifanov and Saakyan, 1982; Hills and Barber, 1986; Maksimovich *et al.*, 1986, 1992; Hills *et al.*, 1990; Yevtushenko and Ukhanska, 1993). The essentially new results, which differ from the solution of this problem and corresponding isothermal problem (see e.g. Galin, 1980) were obtained. So, by Hills and Barber (1986) the existence of the limiting value of the contact patch with increasing total load at constant sliding speed was shown. Moreover, it was found that the violation of solution arose for some combinations of input parameters.

The above authors considered the idealized boundary conditions only: the perfect thermal contact between the bodies and thermoinsulation of free surfaces. The last assumption permits us to use the well known fundamental solution for half-space, which is heated by stationary (see e.g. Carslaw and Jaeger, 1959) or moving (see e.g. Barber, 1984) line heat sources. As a result, the condition of the insulation in the separation region was satisfied automatically and the boundary conditions in the contact region lead to a system of integral equations on the unknown pressure and heat fluxes.

The method of reducing the problem with convective heat transfer from the free surface of contacting bodies to the system of integral equations on the unknown pressure and the contact temperatures as proposed by Yevtushenko *et al.* (1992). In the framework of this approach two limiting cases have been considered: when all of the frictional heat flows into the cylinder (Pauk, 1994) or into the half-space (Grylitsky and Pauk, 1995). In this paper it is supposed that both bodies are elastic and thermally conducting.

STATEMENT OF THE PROBLEM

The model considered is shown in Fig. 1(a). Two bodies are pressed together by force P. One of the bodies (cylinder of radius R) slides on the surface of the other (half-space) and heat is generated due to friction. This ensures that the contact area (a, b) will remain substantially stationary with respect to the cylinder and move at the sliding speed V over the half-space surface. Both bodies are elastic and thermal conductors. Shearing tractions on the interface are assumed to be proportional to the normal pressure, with a coefficient of friction f. In solving the contact problem we take into account the coupling effect between normal and shear tractions.

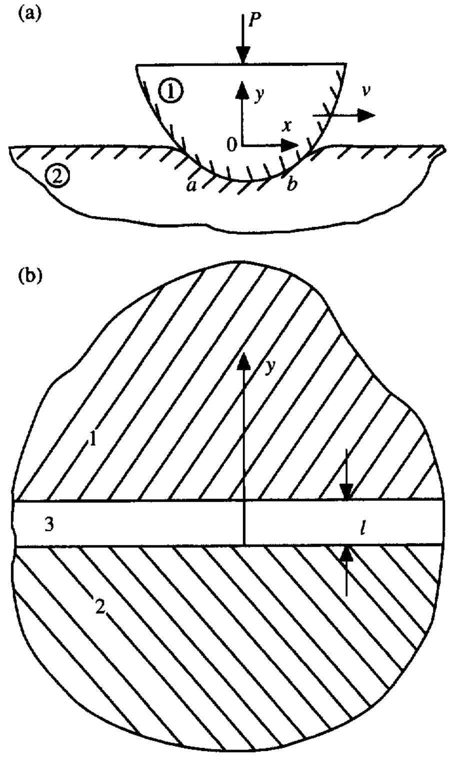

Fig. 1. (a) Geometry of contact. (b) The location of the "third body" between the contact bodies.

In the heat conduction problem we assume that the thermal contact is non-ideal at the interface in the contact region and there is no heat flux across the half-space surface in the separation region. Outside the contact region the convective heat exchange between cylinder and surroundings takes place. We note that the use of the last boundary conditions resolves the classical problem of two-dimensional heat conduction into a half-space with insulated boundaries, in which the temperature does not approach a limit at infinity (see e.g. Carslaw and Jaeger, 1959). With convective cooling at the free surface, a zero limit can be imposed. No such device is needed for the other surface, since the convection of the problem (the heated zone moves over the surface of the half-space) enables us to use a zero initial temperature condition (see e.g. Carslaw and Jaeger, 1959). This is why we use the heat exchange by Newton's law for a cylinder, but not for half-space.

We now introduce rectangular coordinate axes xOy, rigidly connected to the cylinder; in these coordinate axes the contact segment (a, b) is motionless and thermomechanic processes are steady. Values, referred to the cylinder and the half-space, are denoted by the suffices 1 and 2, respectively.

It is well known that heat generation occurs in the thin subsurface layer (see e.g. Bowden and Tabor, 1950). This layer is often called the "third body". The change of thermomechanical properties of this body with the thickness is great because it is composed of the surface roughness, thin inclusions, cracks, etc. Thus, the thermal boundary conditions at the interface in the contact region may be formulated after the solution of the heat conductivity problem for the layer of thickness l with the variable conductivity $K(y)$ [Fig. 1(b)].

In accordance with Podstrigach (1963), we will solve the one-dimensional heat conductivity equation

$$\frac{\mathrm{d}}{\mathrm{d}y}\left[K(y)\frac{\mathrm{d}T(y)}{\mathrm{d}y}\right] = -q(y), \quad 0 \leqslant y \leqslant l \tag{1}$$

with the following boundary condition

$$T = T_1 \quad \text{at}\, y = l, \quad T = T_2 \quad \text{at}\, y = 0 \tag{2}$$

$$K(y)\frac{\mathrm{d}T}{\mathrm{d}y} = -q_1 \quad \text{at}\, y = l, \quad K(0)\frac{\mathrm{d}T}{\mathrm{d}y} = q_2 \quad \text{at}\, y = 0 \tag{3}$$

where T_i are the temperatures, K_i are the conductivities, $q(y)$ is the heat sources function in the layer of the thickness l. We note that $K(l) = K_1$, $K(0) = K_2$.

Integrating eqn (2) and satisfying conditions (3) we obtain

$$q_1 + q_2 = Q \tag{4}$$

where $Q = \int_0^l q(y)\,\mathrm{d}y$.

Now integrating both sides of eqn (1) and taking the conditions (2) into account we find

$$T_1 - T_2 = -\int_0^l q(y)\,\mathrm{d}y \int_y^l \frac{\mathrm{d}\eta}{K(\eta)} + q_2\frac{l}{K_\alpha} \tag{5}$$

where

$$\frac{1}{K_\alpha} = \frac{1}{l}\int_0^l \frac{\mathrm{d}y}{K(y)}. \tag{6}$$

It follows that the upper and lower limits of the difference in the left side of eqn (5) are

$$T_1 - T_2 \leqslant q_2\frac{l}{K_\alpha} \tag{7}$$

$$T_1 - T_2 \geqslant -Q\frac{l}{K_\alpha} + q_2\frac{l}{K_\alpha} = -q_1\frac{l}{K_\alpha}. \tag{8}$$

Since the layer thickness l is very small the mean value of the temperature difference $T_1 - T_2$ may be used. From the inequalities (7) and (8) we have

$$q_2 - q_1 = 2r_0^{-1}(T_1 - T_2) \tag{9}$$

where $r_0 = lK_\alpha^{-1}$ is the contact thermal resistance.

The sum of heat fluxes $q_i(x)$, $i = 1, 2$ entering each oscillating body is equal to the rate of frictional heat generation throughout the contact region (a, b). Hence,

$$q_1(x) + q_2(x) = fVp(x), \quad a \leqslant x \leqslant b \tag{10}$$

where $p(x)$ is the contact pressure.

Hence, the thermal boundary conditions in the contact region are eqns (9) and (10). In the separation region we have

$$K_1\frac{\partial T_1}{\partial y}\bigg|_{y=0} = hT_1\bigg|_{y=0}, \quad K_2\frac{\partial T_2}{\partial y}\bigg|_{y=0} = 0, \quad x < a, x > b \tag{11}$$

where h is the heat transfer coefficient.

The mechanical boundary conditions at $y = 0$ are

$$\sigma_{y1} = \sigma_{y2} = -p(x), \quad a \leqslant x \leqslant b \tag{12}$$

$$\sigma_{xy1} = \sigma_{xy2} = -fp(x), \quad a \leqslant x \leqslant b \tag{13}$$

$$\sigma_{yi} = \sigma_{xyi} = 0, \quad i = 1,2, \quad x < a, x > b \tag{14}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(v_1 - v_2) = -\frac{x}{R}, \quad a \leqslant x \leqslant b \tag{15}$$

where σ_{ij} are the stresses, v_i are the y-direction displacements of bodies.

REDUCTION TO THE SYSTEM OF INTEGRAL EQUATION

The temperature field $T_1(x, y)$ in the half-space due to a stationary heat flux $q_1(x)$ in the region $a \leqslant x \leqslant b$, $y = 0$ and cooling in accordance with the Newton's law (11) in the region $x < a$, $x > b$, $y = 0$ is given by Yevtushenko *et al.* (1992) as

$$T_1(x,y) - \frac{h}{\pi K_1}\int_a^b T_1(x')M_1(x-x',y)\,\mathrm{d}x' = \frac{1}{\pi K_1}\int_a^b q_1(x')M_1(x-x',y)\,\mathrm{d}x' \qquad |x| < \infty, y \geqslant 0 \tag{16}$$

where

$$M_1(x,y) = \int_0^\infty \frac{\cos(\xi x)}{\xi + h/K_1}\exp(-\xi y)\,\mathrm{d}\xi. \tag{17}$$

Here and further we note $T_i(x) \equiv T_i(x, 0)$, $i = 1, 2$.

Body 2 experiences a distribution of moving heat flux. The temperature $T_2(x, y)$ due to the heat flux $q_2(x)$ currently distributed at $a \leqslant x \leqslant b$, $y = 0$ (outside this region the boundary is thermoinsulated) and moving at speed V is given by Carslaw and Jaeger (1959) as

$$T_2(x,y) - \frac{h}{\pi K_2}\int_a^b q_2(x')M_2(x-x',y)\,\mathrm{d}x', \quad |x| < \infty, y \leqslant 0 \tag{18}$$

where

$$M_2(x,y) = \exp(-Vx/2k_2)K_0(V\sqrt{x^2+y^2}/2k_2). \tag{19}$$

k_2 is the thermal diffusivity, $K_0(\cdot)$ is the modified Bessel function of the second kind.

Similarly, the slopes of the surfaces of static and moving bodies due to heat fluxes $q_i(x)$, $i = 1, 2$ are given by Yevtushenko *et al.* (1992) and Barber (1984), respectively, as

$$\frac{\mathrm{d}v_1^{\mathrm{th}}(x)}{\mathrm{d}x} = \frac{\delta_1}{\pi}\int_a^b \left[q_1(x') + hT_1(x')\right]N_1(x-x')\,\mathrm{d}x', \quad |x| < \infty \tag{20}$$

$$\frac{\mathrm{d}v_2^{\mathrm{th}}(x)}{\mathrm{d}x} = -\frac{\delta_2}{\pi}\int_a^b q_2(x')N_2(x-x')\,\mathrm{d}x', \quad |x| < \infty \tag{21}$$

where

$$N_1(x) = \int_0^\infty \frac{\sin(\xi x)}{\xi + h/K_1}\,\mathrm{d}x$$

$$N_2(x) = \begin{cases} \exp(-Vx/2k_2)[I_0(Vx/2k_2) - I_1(Vx/2k_2)], & x > 0 \\ 0, & x < 0 \end{cases}$$

$\delta_i = \alpha_i(1+\nu_i)/K_i$ are the thermal distortions, α_i are the thermal expansions, ν_i are the Poisson's ratios, μ_i are the rigidity moduli, $I_j(\cdot), j = 0, 1$ are the modified Bessel functions of the first kind.

In addition, the surface displacements of the bodies due to interfacial tractions are (see e.g. Galin, 1980)

$$\frac{\mathrm{d}v_i^e(x)}{\mathrm{d}x} = -\frac{1-2\nu_i}{2\mu_i} fp(x) + (-1)^{i+1}\frac{1-\nu_i}{\pi\mu_i}\int_a^b \frac{p(x')\,\mathrm{d}x'}{x-x'}, \quad |x| < \infty, \quad i = 1, 2. \tag{22}$$

Normal displacements on the surface $y = 0$ of the cylinder and semi-space are presented as the superposition of thermoelastic and elastic parts (see e.g. Hills and Barber, 1986)

$$v_i(x) = v_i^{th}(x) + v_i^e(x), \quad i = 1, 2, \quad |x| < \infty \tag{23}$$

where v_i^{th} and v_i^e are given by eqns (20)–(22).

Equations (9) and (10) may be rewritten as

$$q_1(x) = \frac{1}{2} fVp(x) - \frac{1}{r_0}[T_1(x) - T_2(x)], \quad a \leqslant x \leqslant b \tag{24}$$

$$q_2(x) = \frac{1}{2} fVp(x) + \frac{1}{r_0}[T_1(x) - T_2(x)], \quad a \leqslant x \leqslant b. \tag{25}$$

Substituting relations (24) and (25) into (20) and (21), and making use of eqns (23) and (15) gives

$$\begin{aligned} -\left(\frac{1-2\nu_1}{2\mu_1} - \frac{1-2\nu_2}{2\mu_2}\right) fp(x) + \left(\frac{1-\nu_1}{\mu_1} - \frac{1-\nu_2}{\mu_2}\right)\frac{1}{\pi}\int_a^b \frac{p(x')\,\mathrm{d}x'}{x-x'} \\ + \frac{fV}{2\pi}\int_a^b p(x')[\delta_1 N_1(x-x') + \delta_2 N_2(x-x')]\,\mathrm{d}x' \\ + \frac{1}{\pi}\int_a^b T_1(x')[\delta_1(h - r_0^{-1})N_1(x-x') + \delta_2 r_0^{-1} N_2(x-x')]\,\mathrm{d}x' \\ + \frac{1}{\pi}\int_a^b T_2(x')[\delta_1 r_0^{-1} N_1(x-x') - \delta_2 r_0^{-1} N_2(x-x')]\,\mathrm{d}x' = -\frac{x}{R} \quad a \leqslant x \leqslant b. \end{aligned} \tag{26}$$

Two other equations are obtained by substituting expressions (24) and (25) into eqns (16) and (18) at $y = 0$. We obtain

$$\begin{aligned} T_1(x) - \frac{h - r_0^{-1}}{\pi K_1}\int_a^b T_1(x')M_1(x-x')\,\mathrm{d}x' - \frac{r_0^{-1}}{\pi K_1}\int_a^b T_2(x')M_1(x-x')\,\mathrm{d}x' \\ - \frac{fV}{2\pi K_1}\int_a^b p(x')M_1(x-x')\,\mathrm{d}x' = 0, \quad a \leqslant x \leqslant b \end{aligned} \tag{27}$$

$$T_2(x)+\frac{r_0^{-1}}{\pi K_2}\int_a^b T_2(x')M_2(x-x')\,dx'-\frac{r_0^{-1}}{\pi K_2}\int_a^b T_1(x')M_2(x-x')\,dx'$$
$$-\frac{fV}{2\pi K_2}\int_a^b p(x')M_2(x-x')\,dx'=0,\quad a\leqslant x\leqslant b \quad (28)$$

where $M_i(x)\equiv M_i(x,0)$, $i=1,2$.

The load P, applied to the cylinder, will also be prescribed and hence

$$\int_a^b p(x)\,dx=P. \quad (29)$$

To obtain eqns (26)–(29) expressed in terms of dimensionless variables the following notations are introduced

$$x=a_0s+b_0,\quad x'=a_0r+b_0,\quad a_0=(b-a)/2,\quad b_0=(b+a)/2$$
$$\alpha=\frac{1-\nu_1}{\mu_1}+\frac{1-\nu_2}{\mu_2},\quad \beta=\frac{1}{\alpha}\left(\frac{1-2\nu_1}{2\mu_1}-\frac{1-2\nu_2}{2\mu_2}\right),\quad Pe=\frac{Va_0}{2k_2},$$
$$Bi=\frac{ha_0}{K_1},\quad H=\frac{\delta_2k_2}{\alpha},\kappa=\frac{a_0}{r_0K_1},\quad K^*=\frac{K_2}{K_1},\quad \delta^*=\frac{\delta_1}{\delta_2}$$
$$p(x)=\frac{P}{a_0}p^*(s),\quad T_i(x)=\frac{fVP}{K_1+K_2}T_i^*(s),\quad i=1,2. \quad (30)$$

In this way we obtain the system of integral equations

$$-f\beta p^*(s)+\frac{1}{\pi}\int_{-1}^{1}p^*(r)\left\{\frac{1}{s-r}+fPeH[\delta^*N_1(s-r)+N_2(s-r)]\right\}dr$$
$$+\frac{2fPeH}{\pi(1+K^*)}\int_{-1}^{1}T_1^*(r)[\delta^*(Bi-\kappa)N_1(s-r)+\kappa N_2(s-r)]\,dr$$
$$+\frac{2fPeH}{\pi(1+K^*)}\int_{-1}^{1}T_2^*(r)[\delta^*\kappa N_1(s-r)-\kappa N_2(s-r)]\,dr$$
$$=-\left(s+\frac{b_0}{a_0}\right)\Lambda,\quad |s|\leqslant 1 \quad (31)$$

$$T_1^*(s)-\frac{Bi-\kappa}{\pi}\int_{-1}^{1}T_1^*(r)M_1(s-r)\,dr-\frac{\kappa}{\pi}\int_{-1}^{1}T_2^*(r)M_1(s-r)\,dr$$
$$-\frac{1+K^*}{2\pi}\int_{-1}^{1}p^*(r)M_1(s-r)\,dr=0,\quad |s|\leqslant 1 \quad (32)$$

$$T_2^*(x)+\frac{\kappa}{\pi K^*}\int_{-1}^{1}T_2^*(r)M_2(s-r)\,dr-\frac{\kappa}{\pi K^*}\int_{-1}^{1}T_1^*(r)M_2(s-r)\,dr$$
$$-\frac{1+K^*}{2\pi K^*}\int_{-1}^{1}p^*(r)M_2(s-r)\,dr=0,\quad |s|\leqslant 1 \quad (33)$$

$$\int_{-1}^{1} p^*(r)\,\mathrm{d}r = 1. \tag{34}$$

Here

$$\Lambda = \frac{1}{2\pi AB}\left(\frac{P_{\mathrm{H}}}{P}\right),\quad \tan(\pi A) = -\frac{1}{f\beta},\quad 0 < A < 1,\quad B = 1-A.$$

P_{H} is the force necessary to exert the isothermal contact region of the width $2a_0$ (see e.g. Galin, 1980)

$$P_{\mathrm{H}} = \frac{2\pi ABa_0^2}{R\alpha}. \tag{35}$$

The dimensionless kernels of eqns (31)–(33) have the following form

$$M_1(s) = -\mathrm{Ci}(|s|Bi)\cos(sBi) - \mathrm{si}(sBi)\sin(sBi)$$
$$M_2(s) = \exp(-sPe)K_0(|s|Pe)$$
$$N_1(s) = [\mathrm{Ci}(|s|Bi)\sin(|s|Bi) - \mathrm{si}(sBi)\cos(sBi)]\,\mathrm{sign}(s)$$
$$N_2(s) = \begin{cases}\exp(-sPe)[I_0(sPe) - I_1(sPe)], & s > 0\\ 0, & s < 0\end{cases}$$

where si(·) and Ci(·) are sine and cosine integrals, respectively (se e.g. Abramowitz and Stegun, 1964).

We note that the functions $N_i(s)$, $i = 1,2$ are regular and $M_i(s)$, $i = 1,2$ have the logarithmical singularity for small argument.

DISCRETIZATION

Equation (31) is a Cauchy-type singular integral equation with index -1 for an unknown function $p^*(r)$ expressed in terms of $T_i^*(r)$ and certain given functions. Hence, the dimensionless contact pressure $p^*(r)$ may be represented as

$$p^*(s) = \Lambda(1-s)^A(1+s)^B\varphi(s) \tag{36}$$

where $\varphi(s)$ is a bounded continuous function.

The integral eqns (32) and (33) may be considered as Fredholm-type equations of the second kind for unknown functions $T_i^*(s)$, $i = 1,2$. The dimensionless temperatures $T_i^*(s)$ we find in the form $T_i^*(s) = \Lambda T_i^{**}(s)$.

Using the Gauss–Jacobi quadratures by Belocerkovskij and Lifanov (1985) for $p^*(s)$ and replacing the actual distribution of the contact temperatures T_i^* by a piece-wise constant representation, we obtain the discretized form of eqns (31)–(34) as

$$\gamma_{\mathrm{on}} + \frac{1}{\pi}\sum_{k=1}^{n}\varphi(r_k)W_k\left\{\frac{1}{s_m - r_k} + fPeH[\delta^* N_1(s_m - r_k) + N_2(s_m - r_k)]\right\}$$
$$+\frac{2fPeH}{1+K^*}\sum_{k=1}^{n}\frac{2}{n}T_1^{**}(\rho_k)[\delta^*(Bi-\kappa^*)N_1(s_m-\rho_k) + \kappa N_2(s_m-\rho_k)]$$
$$+\frac{2fPeH}{1+K^*}\sum_{k=1}^{n}\frac{2}{n}T_2^{**}(\rho_k)[\delta^*\kappa N_1(s_m-\rho_k) - \kappa N_2(s_m-\rho_k)]$$

$$= -\left(s+\frac{b_0}{a_0}\right), \quad m = 1,2,\ldots n+1 \tag{37}$$

$$T_1^{**}(\rho_m) - \frac{Bi-\kappa}{\pi}\sum_{k=1}^{n}\theta_{km}^{(1)}T_1^{**}(\rho_k) - \frac{\kappa}{\pi}\sum_{k=1}^{n}\theta_{km}^{(1)}T_2^{**}(\rho_k)$$
$$-\frac{1+K^*}{2\pi}\sum_{k=1}^{n}W_k\varphi(r_k)M_1(\rho_m-r_k) = 0, \quad m = 1,2,\ldots n \tag{38}$$

$$T_2^{**}(\rho_m) + \frac{\kappa}{\pi K^*}\sum_{k=1}^{n}\theta_{km}^{(2)}T_2^{**}(\rho_k) - \frac{\kappa}{\pi K^*}\sum_{k=1}^{n}\theta_{km}^{(2)}T_1^{**}(\rho_k)$$
$$-\frac{1+K^*}{2\pi K^*}\sum_{k=1}^{n}W_k\varphi(r_k)M_2(\rho_m-r_k) = 0, \quad m = 1,2,\ldots n \tag{39}$$

$$\sum_{k=1}^{n}W_k\varphi(r_k) = 2\pi AB\left(\frac{P}{P_{\mathrm{H}}}\right). \tag{40}$$

Here

$$W_k = -\frac{2\Gamma(n+A+1)\Gamma(n+B+1)(2n+3)}{(n+1)!\Gamma(n+3)\mathscr{P}_n^{(A,B)'}(r_k)\mathscr{P}_{n+1}^{(A,B)}(r_k)}, \quad k = 1,2,\ldots n$$

$$P_n^{(A,B)}(r_k) \equiv 0, \quad k = 1,2,\ldots n; \quad \mathscr{P}_{n+1}^{(-A,-B)}(s_m) = 0, \quad m = 1,2,\ldots n+1$$

$$\rho_k = -1+2(k-0.5)/n, \quad k = 1,2,\ldots n;$$

$$\theta_{km}^{(i)} = \int_{X_2}^{X_1} M_i(u)\,\mathrm{d}u, \quad i = 1,2.$$

$X_1 = 2(m-k+0.5)/n$, $X_2 = 2(m-k-0.5)/n$, $k, m = 1,2,\ldots,n$, $\mathscr{P}_n^{(A,B)}(\cdot)$ are the Jacobi polynomials, $\Gamma(\cdot)$ is the gamma-function, γ_{on} is the regularized parameter introduced by Belocerkovskij and Lifanov (1985). The solution of the system of linear algebraic eqns (37)–(39) exists provided

$$\lim_{n\to\infty}\gamma_{\mathrm{on}} = 0. \tag{41}$$

Thus, relations (37)–(39) constitute the system of $3n+1$ linear algebraic equations for the same unknown functions γ_{on}, $\varphi(r_k)$, $T_1^{**}(\rho_k)$, $T_2^{**}(\rho_k)$, $k = 1,2,\ldots n$. The difficulties of the solution of this system are connected with the fact that the values a_0 and b_0, and consequently a, b, are also unknown. The algorithm to determine them is described by Yevtushenko *et al.* (1993). When, from eqns (37)–(39) we find all the values, then from eqn (40) we obtain P_{H}/P.

RESULTS AND DISCUSSION

The algorithm described in the previous section was implemented and used to explore the behaviour of the system with various values of the input parameters $n, f, \beta, K^*, \delta^*$ and the independent variables Pe, Bi, κ, a_0/R, b_0/R. The values of β and H for the large range of metals is given by Hills *et al.* (1990). Further, to reduce the number of free variables to manageable proportions, set $f = 0.2$, $\beta = 0.3$, $H = 1$, $\delta^* = 1$ and $\kappa = 0.1$.

Taking the limit (41) into account, the choice of number n permits the condition $|\gamma_{\mathrm{on}}| < 10^{-6}$. In this case, convergence of the numerical scheme was obtained with $n \leqslant 40$, which yields the largest matrix (121×121) which can conveniently be inverted. We also tested the present numerical method with problems which have an analytical solution. For

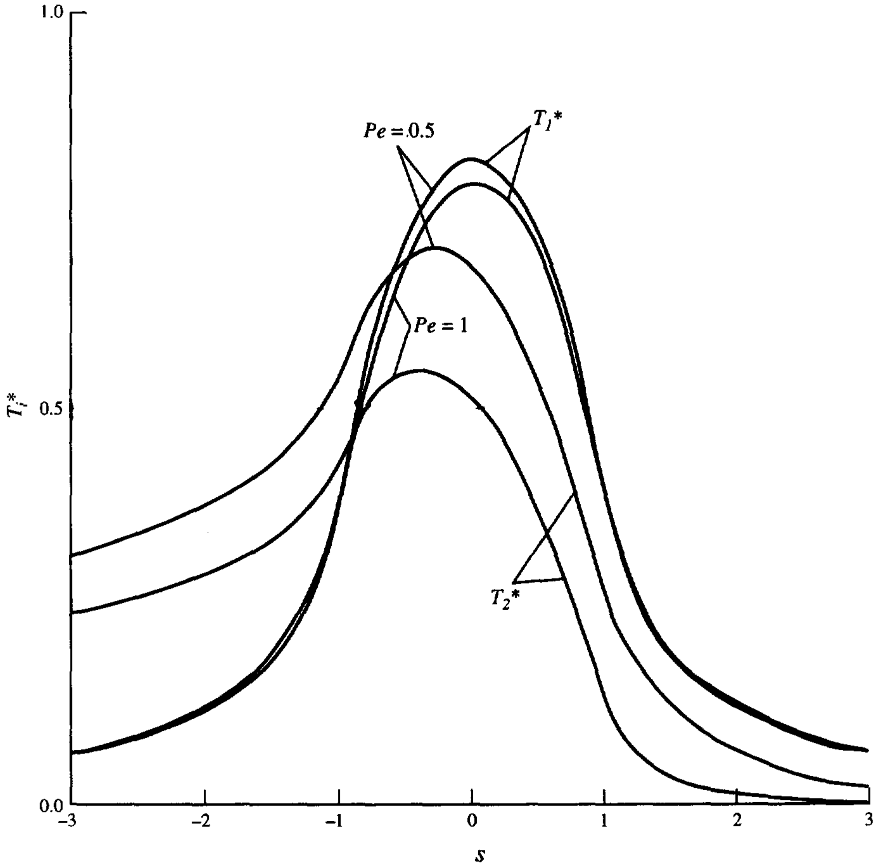

Fig. 2. Distribution of the surface temperatures for $K^* = 1$, $Bi = 1$ and for several values of Peclet's number.

example, the algorithm was validated with closed form solutions of the isothermal plane Hertz contact problem by Galvin (1980) and the solution by Hills *et al.* (1990) with frictional heating where the fast-moving cylinder is a non-conductor. The numerical algorithm gave very good accuracy.

The distribution of dimensionless surface temperature T_i^* of the contact bodies for $Bi = 1$ and for two values of Peclet's number is shown in Fig. 2. The rising of Peclet's number causes a small decrease of stationary temperature of the cylinder and the great falling of quasistationary temperature of the half-space. The jump between temperatures, which is the result of the non-ideal thermal contact, essentially depends on the parameter Pe. This jump increases with increasing Pe. Outside the contact region ($|s| > 1$) the temperatures T_i^* vanish quickly.

We note that the rise of the dimension temperatures T_i [see eqn (30)] is greater at the interface when the sliding speed increases due to great energy generation at the interface by friction. The effect of Pe on the temperature fields (16) and (18) is shown in Fig. 3(a, b). We can see that at low Pe the heat diffusion in the half-space is significant in all directions, but at greater Pe the temperature rising is restricted to a thin subsurface layer. This effect is negligible for the cylinder.

The dependence of P_H/P on the Peclet's number for several values of the dimensionless conductivity K^* is shown in Fig. 4(a). In the limiting case $K^* = 0$ (the half-space is a thermoinsulator), the dependence of P_H/P on Pe is linear in accordance with the equation

$$\frac{P_H}{P} = 1 - \frac{Pe}{3.54}$$

where $Pe = 3.54$ is the limiting value of the Peclet's number at $P \to \infty (P_H/P \to 0)$. In the

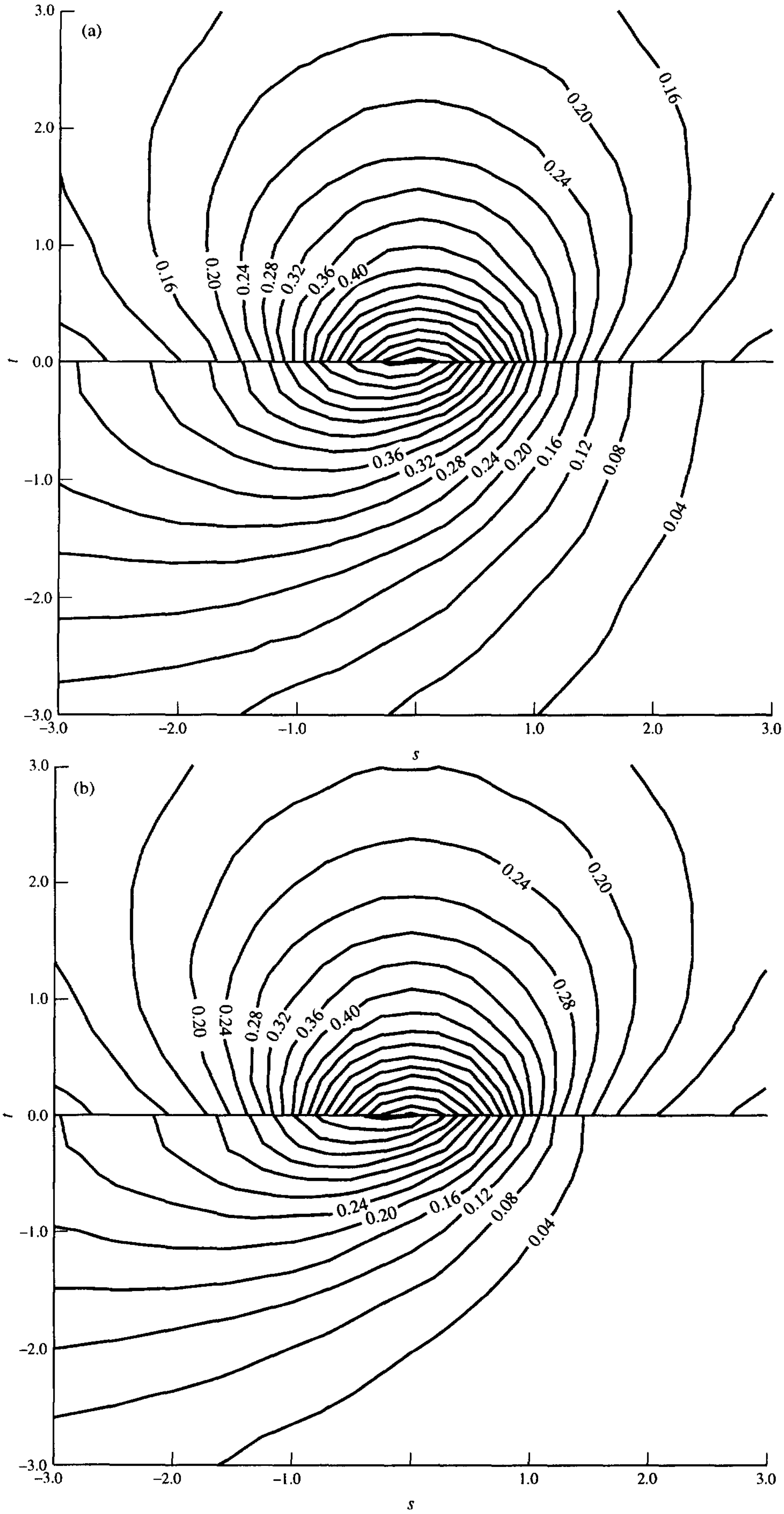

Fig. 3. (a) Temperature contours in bodies for $Bi = 1$ and $Pe = 0.5$. (b) Temperature contours in bodies for $Bi = 1$ and $Pe = 1$.

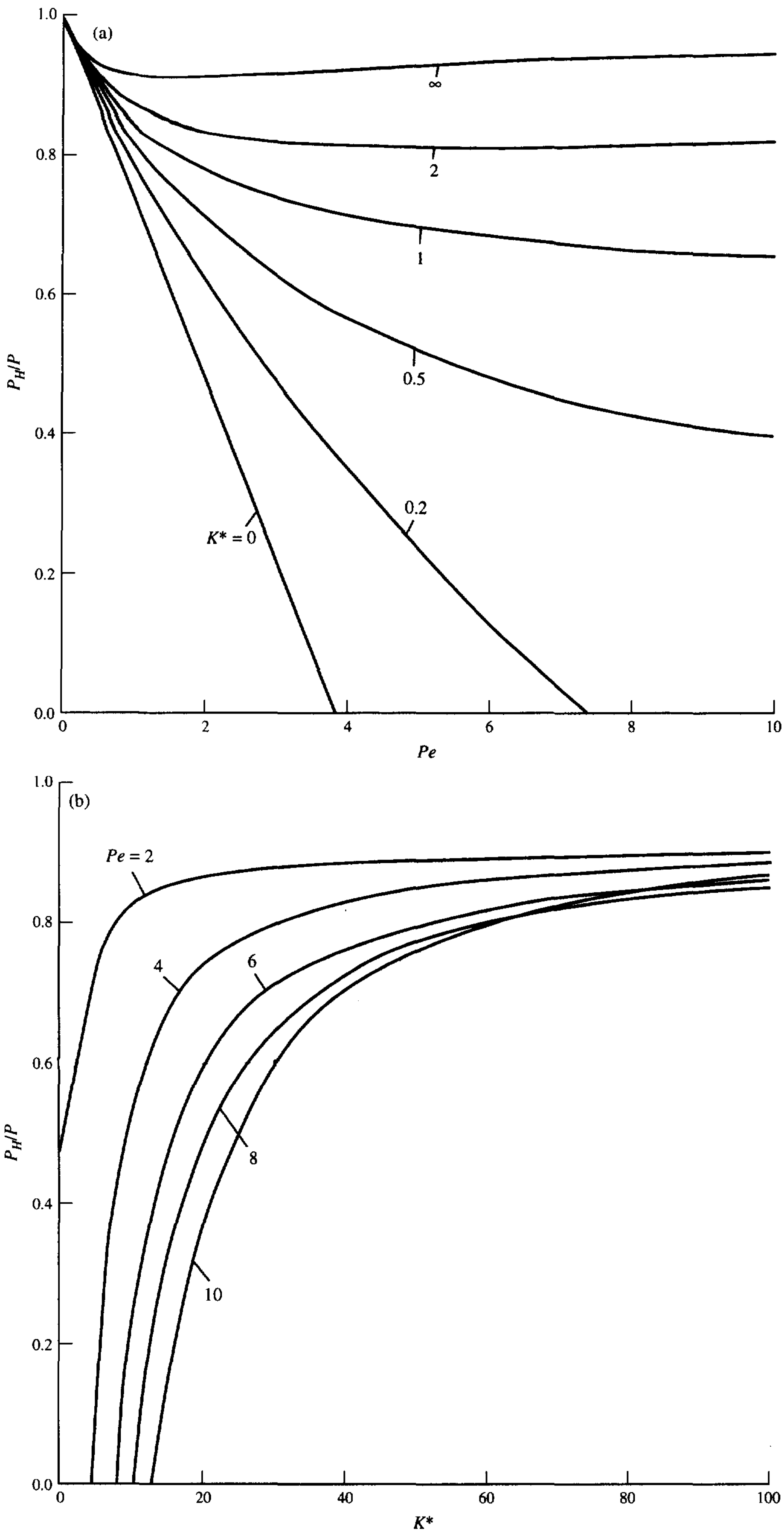

Fig. 4. (a) Dependence of the ratio P_H/P on the Peclet's number for $Bi = 0.1$ and several values of K^*. (b) Dependence of the ratio P_H/P on K^* for $Bi = 0.1$ and several values of the Peclet's number.

cases when both bodies are conductors this dependence is somewhat more complicated. We can see that for $K^* > 1$ the limiting value of Pe does not exist.

The dependence of the ratio P_H/P on K^* in Fig. 4(b) enables the following conclusions to be drawn:

(1) at fixed Pe the ratio P_H/P increases as K^* increases;
(2) P_H/P increases also as Pe decreases;
(3) physically acceptable solutions (i.e. that the interfacial traction in the contact region is non-tensile and that the gap in the separation region is non-negative) are not obtained if simultaneously $Pe \geqslant 4$ and the cylinder is a much poorer conductor than the half-space.

REFERENCES

Abramowitz, M. and Stegun, I. (1964) *Handbook of Mathematical Functions*. National Bureau of Standards.

Barber, J. R. (1984) Thermoelastic displacements and stresses due to a heat source moving over the surface of a half-plane. *Journal of Applied Mechanics* **51,** 636–640.

Belocerkovskij, S. M. and Lifanov, I. K. (1985) *Numerical Methods in Singular Integral Equations*. Nauka, Moscow.

Bowden, F. P. and Tabor, D. (1950) *Friction and Lubrication of Solids*. Oxford University Press, Oxford.

Carslaw, H. S. and Jaeger, J. C. (1959) *Conduction of Heat in Solids*. Oxford University Press, Oxford.

Galin, L. A. (1980) *Contact Problems of Elasticity and Viscoelasticity*. Nauka, Moscow.

Grylitsky, D. V. and Pauk, V. J. (1995) Some quasistationary contact problems for half-space involving heat generation and radiation. *International Journal of Engineering Science* **33,** 1773–1781.

Hills, D. A. and Barber, J. R. (1986) Steady sliding of a circular cylinder over a dissimilar thermally conducting half-plane. *International Journal of Mechanical Science* **28,** 613–622.

Hills, D. A., Nowell, D. and Sackfield (1990) The state of stress induced by cylindrical sliding contacts with frictional heating. *International Journal of Mechanical Science* **32,** 767–778.

Lifanov, I. K. and Saakyan, A. A. (1982) Method of numerical analysis of the problem of indentation of a moving punch into an elastic half-plane involving heat generation. *Prikladnaya Matematica i Mekhanika* **46,** 494–501.

Maksimovich, V. N., Babej, Yu. I., Kratjuk, P. B. *et al.* (1986) Plane thermoelastic contact problem involving frictional heating. *Physicochemical Mechanics of Materials* **3,** 76–81.

Maksimovich, V. N., Kratjuk, P. B., Babej, Yu. I. *et al.* (1992) Influence of the heat generation on the temperature fields and stresses due to frictional reinforced of cylindrical parts. *Physicochemical Mechanics of Materials* **2,** 64–69.

Pauk, V. J. (1994) Plane contact problem involving heat generation and radiation. *Journal of Theoretical and Applied Mechanics* **32,** 829–839.

Podstrigach, Ya. S. (1963) Temperature field in the system of the hard bodies contacting by the thin interface layer. *Engineering Physics Journal* **6,** 129–136.

Yevtushenko, A. A., Pauk, V. J. and Ukhanska, O. M. (1992) The system of integral equations of contact problem on thermal elasticity allowing for quasi-stationary heat generation because of friction. *Dopovidi AN Ukrainy. Ser. A.* **10,** 60–65.

Yevtushenko, A. A. and Ukhanska, O. M. (1993) Plane contact problem of thermoelasticity with quasi-steady frictional heating. *International Journal of Engineering Science* **31,** 1565–1573.